Quantum Malus law for composite systems as a hidden-variable theory

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By referring to only two positions of measuring devices, thus avoiding the counterfactual reasoning of the Bell result, a plausibly general proof is provided that the distribution function of the hidden variables can be neither a continuous nor a singular local function but must be at least a singular nonlocal one. The recognition of Scully and Milonni's theory as a theory which makes the quantum Malus law work for composite systems was the clue to the proof.

Only two loopholes remain in the Bell-like proofs that local hidden variables cannot underlie quantum mechanics (QM). One is the low efficiency of the detectors[1] and the other is a possibility that the measuring process produces a change in the distribution function of the hidden variables for composite systems.[2] The purpose of this paper is to narrow down the latter.

The Bell inequality involves results of at least three incompatible experiments carried out on pairs of subsystems of composite quantum systems and consequently the experiments cannot be carried out jointly (simultaneously). Thus it can be argued that the Bell inequality "excludes only those [hidden-variable theories] in which the measuring process is assumed not to affect the distribution of the hidden variables."[3] In order to match this objection we give a plausibly general proof of the nonlocality of assumed hidden variables which involves only two joint measurements carried out on the subsystems. It is proven that the distribution function $f(\alpha)$ of the hidden variables α can be neither a continuous nor a singular local function but must at least be a singular nonlocal one. The proof is viewed as a reduction of the Clauser-Horne (CH) reformulation of Bell's result.[4] In our search for the reduction, a "new type" of nonlocal hidden-variable theory (HV_3) formulated for electrons by Scully[5] and reformulated for photons by Milonni[6] gave us the clue.

The essential assumption of the Bell-CH theorem is that particular properties (e.g., spin projection) of correlated subsystems of a composite quantum system are at least stochastically predetermined. The theorem itself can therefore be reformulated as follows: "If the measured properties of the subsystems of a composite quantum system are (supposed to be) predetermined, then $f(\alpha)$ must at least be a singular nonlocal function (in order to enable a hidden-variable theory to give the same results as quantum mechanics)."

The aforementioned reduction of the theorem we carry through by specifying the meaning of the "predetermined property" so as to make the theorem read as follows: "If the measured properties of the subsystems are predetermined (prepared) by nature in the same way in which *we* predetermine (prepare) them by our devices (polarizers, Stern-Gerlach devices, etc.), then $f(\alpha)$ must at least be a singular nonlocal function which cannot be considered affected by the measuring process."

For the correlated electrons and photons which we are going to consider, this means that they are assumed to be prepared (predetermined) so as to obey the quantum Malus law (ML) as defined below.

Our elaboration recognizes Scully and Milonni's HV_3 theory as a hidden-variable theory which makes ML work for composite systems. It should be stressed here that Malus' law for composite systems recently put forward by Wódkiewicz[7] as a modification of the (standard) ML applies to the QW amplitudes and therefore has no influence on the present elaboration.

To define ML let us consider individual quantum systems prepared, one by one, along a particular direction which makes an angle α to the vertical. Let them be detected by a detection device (a Stern-Gerlach device for electrons, and an analyzer for photons) deflected at an angle $|\varphi-\alpha|$ with respect to the preparation direction. ML then predicts that the probability (propensity) of confirming the prepared property in the long run is $p(\varphi,\alpha)=\cos^2(\varphi-\alpha)/2C$, where $C=1$ for electrons and $C=\frac{1}{2}$ for photons. Notice that $p(\varphi,\alpha)=\tilde{\pi}_\varphi(\alpha)$, for $C=1$, where $\tilde{\pi}_\varphi(\alpha)$ is given by Eq. (2.4) of Ref. 5.

Let us further consider a system composed of two two-state subsystems which are spontaneously anticorrelatively[8] generated along α. Subscripts 1, 2 of α will refer to one of the two subsystems. Subscripts 1, 2 of φ will refer to one of the two detectors which detect subsystems. Superscripts $+,-$ will refer to spins "up" and "down" for electrons and to "parallel" and "perpendicular" polarizations for photons, respectively. In the sequel we shall call these states positive and negative, respectively. Of course, α^- will never appear for photons since it is devoid of a physical meaning in this case.

The probability of detecting one of the subsystems in its positive state *and* the other in its negative state is for electrons

$$p(\varphi_1^+,\alpha_1^+;\varphi_2^-,\alpha_2^-)=\cos^2\frac{\varphi_1-\alpha}{2}\cos^2\frac{\varphi_2-\alpha}{2}$$

if the first subsystem is "prepared" in the positive (along α) and the second, anticorrelatively, in the negative state, and

$$p(\varphi_1^+,\alpha_1^-;\varphi_2^-,\alpha_2^+)=\sin^2\frac{\varphi_1-\alpha}{2}\sin^2\frac{\varphi_2-\alpha}{2}$$

if it is the other way around. The other probabilities are defined in an analogous way. For photons, e.g.,

$$p(\varphi_1^+,\alpha_1^+;\varphi_2^+,\alpha_2^+)=\cos^2(\varphi_1-\alpha)\cos^2(\varphi_2-\alpha) \ .$$

These probabilities are in Refs. 5 and 6 expressed by means of the δ functions in Eqs. (4.7) and (1), respectively. For example,

$$p(\varphi_1^+,\alpha_1^+;\varphi_2^+,\alpha_2^+) = \int \delta(\alpha-\beta)\cos^2(\varphi_1-\alpha)\cos^2(\varphi_2-\beta)d\beta \ .$$

The probability of joint detection of a positive state by the first detector *and* a negative by the second for electrons is in general given by the expression

$$P(\varphi_1^+,\varphi_2^-)=\int_0^{2\pi} f(\alpha)\tfrac{1}{2}[p(\varphi_1^+,\alpha_1^+;\varphi_2^-,\alpha_2^-) + p(\varphi_1^+,\alpha_1^-;\varphi_2^-,\alpha_2^+)]d\alpha \ . \quad (1)$$

If we now abandon any knowledge of a particular orientation of α however assuming that particular though unknown orientations do take place and if we assume that the orientations do not depend on either φ_1 or φ_2, then we can always, up to a desirable precision, approximate the appropriate continuous density function $f(\alpha)$ by the series $\sum_{i=0}^{n} C_i \cos^i\alpha$. After some calculation we get

$$P(\varphi_1^+,\varphi_2^-)=D\cos^2\frac{\varphi_1-\varphi_2}{2}+E\cos(\varphi_1+\varphi_2)+F \ ,$$

where

$$D=\sum_i C_{2i}\frac{(2i-1)!!}{(2i)!!} \ ,$$

$$E=\sum_i C_{2i}\frac{(2i^2+2i-1)(2i-1)!!}{2(2i+4)!!} \ ,$$

$$F=\sum_i C_{2i}\frac{(2i-1)!!}{2(2i)!!} \ .$$

The only way to obtain

$$P(\varphi_1^+,\varphi_2^-)=\frac{1}{2}\cos^2\frac{\varphi_1-\varphi_2}{2}=P_{\mathrm{QM}}(\varphi_1^+,\varphi_2^-) \ ,$$

which is the QM result, is to assume $D=\frac{1}{2}$ and $E=F=0$. However, this leads to a contradiction since $2F=D$. The same result follows for the other probabilities for electrons and photons.

If we assume some particular, known or unknown singular value for α, let us say $\alpha=\psi$, then $P(\varphi_1,\varphi_2)$ can be expressed by assuming $f(\alpha)=\delta(\alpha-\psi)$. Thus we obtain $P(\varphi_1,\varphi_2)\neq P_{\mathrm{QM}}(\varphi_1,\varphi_2)$ as in the previous case.

Hence $P(\varphi_1,\varphi_2)\neq P_{\mathrm{QM}}(\varphi_1,\varphi_2)$ no matter which $f(\alpha)$ we choose so far as it does not depend on φ_1 and φ_2. Therefore both Scully and Milonni are "motivated" to consider, in effect, the following nonlocal choice: $f(\alpha)=\delta(\alpha-\varphi_1)$ which gives $P(\varphi_1,\varphi_2)=P_{\mathrm{QM}}(\varphi_1,\varphi_2)$. Namely, if we introduce $f(\alpha)=\delta(\alpha-\varphi_1)$ into the equation for electrons for $P(\varphi_1^+,\varphi_2^+)$ [which is the analogue of Eq. (1)], we obtain

$$\begin{aligned} P(\varphi_1^+,\varphi_2^+) &= \int \delta(\alpha-\varphi_1)\frac{1}{2}\left[\cos^2\frac{\varphi_1-\alpha}{2}\sin^2\frac{\varphi_2-\alpha}{2}+\sin^2\frac{\varphi_1-\alpha}{2}\cos^2\frac{\varphi_2-\alpha}{2}\right]d\alpha \\ &= \int \tfrac{1}{2}[\delta(\alpha-\varphi_1)+\delta(\alpha-\varphi_1-\pi)]\sin^2\frac{\varphi_1-\alpha}{2}\cos^2\frac{\varphi_2-\alpha}{2}d\alpha \end{aligned} \quad (2)$$

which is nothing but $\tilde{\mathcal{P}}(\varphi_1,\varphi_2)$ given by Eq. (4.7) of Ref. 5, as can be easily calculated. In other words, although being structurally different,[6,9] HV_3 and QM predict the same experimental outcomes; i.e., they give the same final results in the sense of $P(\varphi_1,\varphi_2)=P_{\mathrm{QM}}(\varphi_1,\varphi_2)$.

In conclusion, if one wanted to formulate a local hidden-variable theory which presupposes that the measuring process affects the distribution function of the hidden variables, one would have to start with the assumption that nature prepares quantum systems in a different way than we do.

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